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Mini-max Regret Criterion in Patient Therapeutic Healthcare Delivery

Nzerem, Francis Egenti[1*] & Orumie, Ukamaka Cynthia[2]

[1,2]*Department of Mathematics and Statistics, University of Port Harcourt, Nigeria. Email: frankjournals@yahoo.com**

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ABSTRACT

Medics are most often on the horn of a dilemma; they either make the right decision or live in *regret.* The evasion of the "we did our best" syndrome may be effectuated when a judicious choice is made with admissible constraints. The risk involved in decision-making is so inscrutable in situations that may lead to irreversibility as are observable in healthcare delivery. The *Mini-max regret criterion* depicts the benchmark required of a decision-maker (the caregiver) in cushioning the deleterious effect of taking a less optimal decision, possibly, in the presence of a better option. This work, while presenting such criterion, considered two treatment options-*status quo* (i.e. usual/existing) option and the *inventive* (innovative) option. The options were applied to effect both individualized and fractional treatments. The bound for the worst-case *regret* was furnished.

Keywords: Optimization; Decision maker; Medical; Penalty function; Feasibility.

1. Introduction

The health sector, just like the other sectors of life, has its challenges and singularities. Many maladies present phenomenal issues that elicit adroit medical decisions. A major setback that appears frequently in sound healthcare delivery is an inadmissible decision made by the healthcare deliverer in the event of treatment. The consequence of *in good faith* wrong decision may include maiming and death. There is a need to minimize the dark sides of decision-making if they cannot be averted. This calls for the applicability of optimization methods to healthcare problems.

Many constrained optimization methods have been applied in health care services and medical supplies [1, 2, 3]. Such methods extend to pharmaco-economics (Earnshaw and Dennett [4]), HIV prevention (Zaric and Brandeau [5], Lasry *et al*.[6]). It is echoed in Zirkelbach *et al*. [7] that drug dose optimization processes are aimed at minimizing toxicity as well as maximizing benefits to patients. Interestingly, Crown *et al*. [8] identified, among other things, "the types of problems for which optimal solutions can be determined in real-world health applications", and "the appropriate optimization methods for these problems." As good and informing as the aforesaid optimization details are, they fall short of incorporating or projecting the pitfalls when a decision *goes wrong*. This is the bastion of the so-called *mini-max* optimization modelling. The mini-max decision criteria have been applied to varied life endeavours [9, 10]. Among the very incisive works is the one, dubbed here for brevity, "a game against nature", by Ulansky and Raza [11]. Interestingly, Daskin and Hesse [12] tactfully modelled strategic facility location using P-mini-max regret method, albeit not in the aspect of patient handling. 'Regret' is an unassailable component of ex-post decision-making.

The question of the *ex-ante* and *ex-post* magnitude of risk in decision-making is so inscrutable in a scenario that involves such irreversibility as maiming or death. Oftentimes the decision maker (DM, here the Medic) may feel 'regret' if they make a wrong decision in the dispensation of treatment. They therefore must take the projected regret into cognition in decision-making. In a noble medical practice, the DM need to project the possible inputs

Asian Journal of Basic Science & Research
Volume 5, Issue 1, Pages 109-118, January-March 2023

(especially the type, quality, quantity, time, facilities, and routes of drugs) required for a given presentation. In a bid to hedge against the deleterious impact of some inputs the *mini-max* (regret) criteria are often used to furnish conservative decisions (see Rosenhead *et al*. [13], Kouvelis and Yu[14]). In the mini-max regret criterion, the decision-making is essentially evaluated *ex-post*. In some cases, it is suggested by Kouvelis and Yu [14] that *maximum regret* can indicate how much the performance of a decision may be improved if all uncertainties/inaccuracies could be resolved. In this work, the *mini-max regret* criterion is employed in the decision-based dispensation of medical treatment.

2. Mini-max regret criterion

The *mini-max criterion* is seen as conservative because it does not consider missed opportunities. The *min-max regret* criterion is appropriate in circumstances where the decision maker may feel regret if they make a wrong decision. They, therefore, take the projected regret into account when making decisions. A maximum regret can be an indicator of how much the performance of a decision can be improved if all uncertainties/fuzziness are resolvable [14]. The min-max regret criterion is applicable in situations where decisions are analysed ex-post. Mini-max and mini-max regret criteria are often considered reference criteria and a basis for robustness analysis. The mini-max and mini-max regret criteria are sometimes considered inappropriate since they are too pessimistic for decision-makers who are eager to accept some degree of risk [10]. The added resentment against their use is their inclination to worst-case scenarios which are at times unlikely to occur. However, the health sector should not give chances such improbability, hence the need to apply the mini-max regret criterion.

2.1. Optimization of Utility Functions

Many phenomena are analogous to occurrences in nature. They may be modelled as an aspect of game theory. Suppose $\boldsymbol{X} = \{X_1, X_2, \ldots, X_n\}$ is a finite set of attributes with finite domains. Define a *state* as an assignment $\mathbf{x} \in Dom(\boldsymbol{X})$. The attributes, which may be Boolean (Boutilier [15]), have a set of constraints C_l(l=1,…, L) over them. Let $Feas(\boldsymbol{X})$ encode the subset of feasible *states* (i.e., decisions/assignments that satisfy C).

Let us assume there is a known utility function $u: Dom(\boldsymbol{X}) \rightarrow \boldsymbol{R}$. We seek an optimal feasible state $\mathbf{x}^*$ (decision) such that

$$\mathbf{x}^* \in \arg\max_{\mathbf{x}\in Feas(\boldsymbol{X})} u(\mathbf{x}) \cdot \qquad (1)$$

Problem (1) may be presented as a linear integer programming problem in the form

$$\max_{\{B_{\mathbf{x}}X_i\}} \sum_{\mathbf{x}} u_{\mathbf{x}} B_{\mathbf{x}} \qquad (2)$$
$$\text{subject to } \mathrm{C}_1 \text{ and } C_2$$

where (see [15]):

(i) Variables $B_{\mathbf{x}}$ are such that for each $\mathbf{x} \in Dom(\mathbf{X})$, $B_{\mathbf{x}}$ is a Boolean variable indicating whether $\mathbf{x}$ is the state chosen (decision made).

(ii) Variables X_i are such that X_i is a 0-1 variable corresponding to the i-th attribute.

(iii) Coefficients $u_{\mathbf{x}}$ are such that for each $\mathbf{x} \in Dom(\boldsymbol{X})$, constant $u_{\mathbf{x}}$ denotes the (known) utility of state $\mathbf{x}$.

(iv) Constraint set C_1 is such that for each variable $B_{\mathbf{x}}$, a constraint that relates it to its corresponding variable assignment is imposed. Specifically, for each $\boldsymbol{X}_i$: if $\boldsymbol{X}_i$ is true in $\mathbf{x}$, we constrain $B_{\mathbf{x}} \leq \boldsymbol{X}_i$; and if $\boldsymbol{X}_i$ is false in $\mathbf{x}$, we constrain $B_{\mathbf{x}} \leq 1 - \boldsymbol{X}_i$. We denote by C_1 these constraints.

(v) Constraint set C_2 is such that each feasibility constraint C_l is imposed on the attributes $\boldsymbol{X}_i \in \mathbf{X}[l]$

2.2. Some salient mini-max criterions

Blackwell and Girshick [16] postulated that at any given time, only one of n possible states $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ that are collectively exhaustive and mutually exclusive is available to an attribute. The states set $\{\mathbf{x}_1,\ldots,\mathbf{x}_n\}$ is generated by DMs based either on assumptions or experience. We suppose that the DM has a set of decisions $\{\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots, \mathbf{x}_r^*\}$ regarding the attributes. Decision-making is said to be taken *under risk* (applicable under Bayes criterion [15]) when the probabilities of the states $P(\mathbf{x_1})$, $P(\mathbf{x_2})$, …, $P(\mathbf{x_n})$ are known otherwise, decision-making is *under ignorance* (Wald [17] and Savage criteria [18]).

Let the payoff matrix, $A = \|A_{ij}\|_{\substack{i=1,\ldots,m \\ j=1,\ldots,n}}$ and the risk matrix, $R = \|R_{ij}\|$ be known. The expression for R_{ij} is determined by

$$R_{ij} = A_{ij} - \min_{i=1,\ldots,m} A_{ij} \cdot \tag{3}$$

where A_{ij} is the payoff related to the decision x_i^* and state of the $\mathbf{x}_j$. Thus, the term R_{ij} is the variance between the payoff that the DM might obtain with the i-th decision and the probable outcome if they knew the actual state of the occurrence. Considering Bayes's decision for minimizing the average risk we have **[18]**

$$\xi_g^{opt} \Rightarrow \sum_{j=1}^{n} R_{g,j} P(\mathrm{x}_j) = \min_{i=1,\ldots m} \left[\sum_{j=1}^{n} R_{i,j} P(\mathrm{x}_j) \right]. \tag{4}$$

From the standpoint of the Wald criterion, the worst scenario owing to lack of information may be conciliated by choosing a suitable mini-max criterion; if the matrix component A_{ij} encodes the DM's payoff then a decision that placates the mini-max value is taken such that

$$\xi_g^{opt} \Rightarrow \min_{i=1,\ldots,m} \max_{j=1,\ldots,n} A_{ij}, \tag{5}$$

which is analogous to the Savage minimax *risk* criterion

$$\xi_g^{opt} \Rightarrow \min_{i=1,\ldots,m} \max_{j=1,\ldots,n} R_{ij} \cdot \tag{6}$$

2.3. Mini-max regret

We now emphasize the situation in which the utility function is unknown. The unspecified nature of the function precludes its maximization. Nonetheless, the imposition of constraints, in the form of bounds, can make its optimization tractable. The mini-max criterion chooses the assignment, $\mathbf{x}$ that furnishes minimum max-regret. The

Asian Journal of Basic Science & Research
Volume 5, Issue 1, Pages 109-118, January-March 2023

max-regret is the biggest quantity by which one could "regret" choosing action **x**, while letting the utility function vary within the bounds. Considering the discrete scenario, the mini-max (regret) form related to a minimization problem $M \in C$ consists of a finite set S of scenarios as input, where each scenario $s \in S$ is represented by a vector $c_i^s = (c_1^s, \dots, c_n^s)$, with $c_i^s \in \square; i = 1, \dots, n$.

Let $\delta(\mathbf{x}, s) = \sum_{i=1}^{n} c_i^s x_i$ be the value of solution $\mathbf{x} \in X$ under scenario s ∈ S, $\mathbf{X}_s^*$ an optimal solution under scenario *s*, and $\delta_s^* = \delta(\mathbf{x}_s^*, s)$ the associated optimal value. The mini-max form corresponding to *M* is seeking a solution that has the *best worst-case* value through all scenarios, given by: 11112

$$\min_{x \in X} \max_{s \in S} \delta(\mathbf{x}, s). \qquad (7)$$

For a given solution $x \in X$ the regret, $R(\mathbf{x}, s)$, under scenario s ∈ S is reads:

$$\mathcal{R}(\mathbf{x}, s) = \delta(\mathbf{x}, s) - \delta_s^* \quad \text{(cf. equation (3)).} \qquad (8)$$

In essence, the *maximum regret* R_{max} reads

$$\mathcal{R}_{\max}(\mathbf{x}) = \max_{s \in S} \mathcal{R}(\mathbf{x}, s). \qquad (9)$$

The problem of minimizing the maximum regret is therefore in the form:

$$\min_{x \in X} \mathcal{R}_{\max}(\mathbf{x}) = \min_{x \in X} \max_{s \in S} (\delta(\mathbf{x}, s) - \delta_s^*). \qquad (10)$$

The mini-max regret criterion is justified in some adroit considerations; in the event of uncertainty or imprecision in a decision, it is expedient to find a solution with performances reasonably close to the optimal values under all scenarios. Thus, a threshold ε may be set and a solution **x** ∈ X sought such that $\delta(\mathbf{x}, s) - \delta_s^* \le \varepsilon$ for all s ∈ S. Also, **x** should satisfy $R_{max}(\mathbf{x}) \le \varepsilon$. A search for such a solution with ε as small as possible amounts to determining a mini-max regret solution [20].

3. Patient therapeutic health delivery

This section considers the application of mini-max criterion to a patient's healthcare delivery.

The criterion proposes that the DM observes the maximum regret of each approach and selects the one with the least value. The health sector DMs must demonstrate adequate caution towards ensuring that the selected choice must fare well/better when compared to each other alternatives notwithstanding what situation arises. Medics are most likely aware that *regret* is a negative disposition displayed (sub) consciously when learning that an alternative course of action (i.e. not taken) would have yielded a more favourable outcome.

3.1. The Decision-making

Assume there exists a treatment population *Q*, with the members q ∈ *Q*. The population's probability space is (Q; Σ_q; P); $P(q) = 0$ for all *q*. A DM is required to assign one of two treatments T ∈ {0; 1} to each member *q*. Define a response function of each member of the treatment population by $y^q(t)$: {0; 1} → Y. Thus, the function maps treatments onto outcomes. If it is assumed that the members of *Q* are indistinguishable, as specified by Stoye [21],

(see Manski and Tetenov [22], for the major part of this section) allocating treatment t evokes a random variable Y_t (the potential outcome) with distribution $P(y^q(t))$. Any feasible treatment rule either allots each person to one treatment (see Quian and Murphy[23]) or fractionally distributes persons across the treatments. For a singleton rule **x**∈Feas(X), for some t ∈ T, we have

$$[\mathbf{x}(t)=1, \mathbf{x}(t')=0 \quad \forall t' \neq t]. \tag{11}$$

For each rule x, let $U(\mathbf{x},P) \equiv \sum_{t\in T} \mathbf{x}(t).\mathrm{E}[u(t)]$ encode the mean value of u realized under **x**. The DM is faced with the problem

$$\max_{\mathrm{x}\in\mathrm{Feas(X)}} f[U(\mathbf{x},P)]. \tag{12}$$

(Note that $f(.)$ is strictly increasing). By the optimal rule, we get

$$f^*(P) \equiv f\{(\max_{t\in T}\mathrm{E}[u(t)]\}. \tag{13}$$

Let G encode the feasible states of nature. Thus, the feasible values of P are in (P_η, η ϵ G); the feasible values for {{E[$u(t)$], t ϵ T} are {{E [$u(t)$], t ϵ T]}, η ϵ G }.

In the main, we assume that DM is faced with treatment options in the absence of a dominant treatment. Two treatment possibilities exist: $t = \tau_1$ encoding the *status quo* (i.e. usual/existing) treatment and t = τ_2 encoding *inventive* treatment. If the whole patients were to receive one treatment the mean outcomes are $\gamma \equiv \mathrm{E}[u(\tau_1)]$ and $\varphi = \mathrm{E}[u(\tau_2)]$ respectively. The DM has knowledge of γ and that φ ϵ (φ_η, η ϵ G) of possible mean outcomes.

Let a rule assign a fraction ψ of the population to treatment τ_2 and the remaining $1-\psi$ to treatment τ_1. The mean outcome under this rule is:

$$\tau_1(1-\psi)+\tau_2\psi = \tau_1 + (\tau_2-\tau_1)\psi\,. \tag{14}$$

Thus, the generalized treatment is $f(\tau_1 + (\tau_2-\tau_1)\psi)$. The DM's choice when γ is identified and φ ϵ (φ_η, η ϵ G) is of interest. The minimax-regret criterion given by

$$\inf_{\mathrm{x}\in\mathrm{Feas(X)}} \sup_{\eta\in G} f^*(P_\eta) - f[U(\mathrm{x}, P_\eta)] \tag{15}$$

employs only the fact that the state of nature lies in G. $f^*(P) \equiv f\{(\max_{t\in T}\mathrm{E}_\eta[u(t)]\}$ encodes the optimal generalized treatment attainable if it were revealed that $P=P_\eta$. The lack of knowledge of the actual state of nature may result in the loss of optimal generalized treatment that leads to the *regret*

$$\mathrm{R}(\mathbf{x}) = f^*(P_\eta) - f[U(\mathrm{x}, P_\eta)] \tag{16}$$

when rule **x** is chosen.

Let two treatments $T = \{\tau_1, \tau_2\}$ be available. These evoke *fractional* minimax-regret problems:

$$\inf_{\psi\in[0,1]} \sup_{\eta\in G} \max\{f\{E_\eta[u(\tau_1)]\}, f\{E_\eta[u(\tau_2)]\}\} - f\{(1-\psi)E_\eta[u(\tau_1)] + \psi E_\eta[u(\tau_2)]\} \tag{17}$$

We consider an individualized decision a DM made on a subject and, in line with Yifan [24], the decision rule is a mapping from the covariate space to the binary action space χ ϵ{−1, 1}. If A_v is a likely outcome of a DM's action

under an intervention that puts χ to a value v, $A_{D(X)}$ is the likely outcome under an assumed intervention that assigns χ according to the rule D; thus:

$$A_{D(X)} \equiv A_1 I\{D(X)=1\} + A_{-1} I\{D(X)=-1\}, \tag{18}$$

where $I\{.\}$ encodes the indicator function; $E[A_{D(X)}]$ encodes the value function [23].

We consider some positivity assumptions (see Yifan [24]): A DM's perceived treatment outcome may match their likely outcome under a specified decision rule when the realized action matches their probable assignment under the rule. In this scenario, $A = A_{D(X)}$ when $\chi = D(X)$ is almost sure; the next is to assume for any perceived covariates X, the DM has a chance to take either action. Thus, $P(\chi = v|X) > 0$ for $v = \pm 1$ almost surely. The optimal decision rule D^* that holds well the illustration is of the form:

$$D^*(X) = sign\{E(A_1 - A_{-1} \mid X)>0\} \text{ or } D^* = \arg\max_D E[A_{D(X)}]\cdot \tag{19}$$

Let $L_{-1}(X)$, $U_{-1}(X)$, $L_1(X)$, $U_1(X)$ encode lower and upper bounds for $E(A_{-1}|X)$ and $E(A_1|X)$; henceforth, we let lower and upper bounds for $E(A_1 - A_{-1}|X)$ be in the form $L(X) = L_1(X) - U_{-1}(X)$ and $U(X) = U_1(X) - L_{-1}(X)$. An optimistic DM would undertake the programme that entails $\max_D \max E[A_{D(X)}]$ or ostensibly

$$\max_D E[E(A_{-1}|X)+\mathcal{L}(X)I\{D(X)=1\}], \tag{20}$$

With the associated rule:

$$D(h(x),x) = \begin{cases} 1 & u_1(x) > u_{-1}(x), \\ -1 & u_1(x) < u_{-1}(x). \end{cases} \tag{21}$$

In the above, $h(x)$ encodes the (preferred) action index; u_1 and u_{-1} encode the upper and lower index of $\mathbf{x}$ respectively.

In the event of uncertainty, the mini-max regret (opportunist) criterion is invoked. The corresponding rule takes the form

$$D(h(x),x) = \begin{cases} 1 & \mathcal{L}(x) > 0 \text{ or } \mathcal{L}(x) < 0 < u(x), |u(x)| > |\mathcal{L}(x)|, \\ -1 & \mathcal{L}(x) < 0 \text{ or } \mathcal{L}(x) < 0 < u(x), |u(x)| < |\mathcal{L}(x)|. \end{cases} \tag{22}$$

In the case above $h(x) = ½$. Note that for a rule D, with a defined value function $E[A_{D(X)}]$ the regret is $E[A_{D^*(X)}] - E[A_{D(X)}]$.

In the mixed portfolio strategy, the opportunist DM may not put all of their eggs in one basket. Suppose, in this context, $P(\mathbf{x})$ denotes the probability of taking $\chi = 1$ given $X = x$; by the definition of the mini-max regret criterion, the following problem for $P(x)$ is posed (Yifan [23]):

$$\min_{P(x)} \max([1-P(x)]\max\{U(x),0\}, P(x)\max\{-\mathcal{L}(x),0\}), \tag{23}$$

With the solution

Asian Journal of Basic Science & Research
Volume 5, Issue 1, Pages 109-118, January-March 2023

$$P^{*}(x)=\begin{cases}1 & \mathcal{L}(x)>0\\ 0 & U(x)<0\\ \dfrac{U(x)}{U(x)-\mathcal{L}(x)} & \mathcal{L}(x)<0<U(x).\end{cases} \tag{24}$$

The choice of $P^*(x)$ above ensures the worst-case regret is not greater than

$$\begin{cases}0 & U(x)<0 \text{ or } \mathcal{L}(x)>0,\\ -\dfrac{\mathcal{L}(x)U(x)}{U(x)-\mathcal{L}(x)} & \mathcal{L}(x)<0<U(x).\end{cases} \tag{25}$$

Suppose there exist pairwise decision options ($\mathbf{x}, \mathbf{x}_1$) from which a DM expects a desired outlook. The regret of choice follows. For each factor l^k and pair $\mathbf{x}[k]$, $\mathbf{x}_1[k]$ the local pairwise regret is such that $r_{\mathbf{x}[k],\,\mathbf{x}1[k]} = S_{\mathbf{x}1[k]} - S_{\mathbf{x}[k]}$ when $\mathbf{x}[k] \neq \mathbf{x}_1[k]$ and $r_{\mathbf{x}[k],\,\mathbf{x}1[k]} = 0$ when $\mathbf{x}[k] = \mathbf{x}_1[\mathrm{k}]$. The maximum regret over the aggregated local pairwise regrets takes the form

$$\mathcal{R}_{\max}(\mathbf{x}, s \in S) = \max_{\mathbf{x}_1 \in Feas(X_1)} \sum_k r_{\mathbf{x}[k],\mathbf{x}_1[k]}, \tag{26}$$

Noting that

$$\mathcal{R}(\mathbf{x}, \mathbf{x}_1, s \in S) = \sum_k r_{\mathbf{x}[k],\mathbf{x}_1[k]}. \tag{27}$$

Maximizing the regret leads to

$$\mathcal{R}_{\max}(\mathbf{x}, s \in S) = \max_{\{L_{\mathbf{x}_1[k]}, X_{(1)i,}\}} \sum_k \sum_{\mathbf{x}_1[k]} r_{\mathbf{x}[k],\mathbf{x}_1[k]} L_{\mathbf{x}_1[k]} \text{ subject to } C_1 \text{ and } C_2 \tag{28}$$

The constraints in (12**)** hold well for C_1 and C_2. The associated mini-max regret mixed integer programme is

$$\begin{aligned}\min_{\mathbf{x}\in\mathbf{X}} \mathcal{R}_{\max}(\mathbf{x}, s \in S) &= \min_{\{L_{\mathbf{x}_1[k]}, X_{i,}\}} \max_{\mathbf{x}_1 \in Feas(X_1)} \sum_k \sum_{\mathbf{x}_1[k]} r_{\mathbf{x}[k],\mathbf{x}_1[k]} B_{\mathbf{x}[k]} \text{ subject to } C_1 \text{ and } C_2\\ &= \min_{\{L_{\mathbf{x}[k]}, X_{i,}\}} W\end{aligned} \tag{29}$$

$$\text{subject to} \begin{cases} W \geq \sum_k \sum_{\mathbf{x}[k]} r_{\mathbf{x}[k],\mathbf{x}_1[k]} B_{\mathbf{x}[k]} & \forall \mathbf{x}_1 \in Feas(\mathbf{X}_1)\\ C_1 \text{ and } C_2 \end{cases} \tag{30}$$

where W relates to the maximum regret of any state. In (28) the variables for the minimization were presented, and in (29) the minimax program is converted to a minimization program (see [15]) The programme (29) consists of one constraint per feasible state $\mathbf{x}_1$, therefore it is not largely compact.

3.2. Penalizing feasibility constraints

An effective way of de-constraining the minimax problem is by introducing the penalty function(s) (see for instance Li and Pan [25]). Now consider our problem (28) and the constraint (29). Suppose some penalty terms $\xi\mathbf{x}[l]$ are imposed for each constraint C_l. We have in line with [15]

$$\min_{\mathbf{x}\in\mathbf{X}} \mathcal{R}_{\max}(\mathbf{x}, s \in S) = \min_{\{L_{\mathbf{x}[k]}, X_i, W\}} W \tag{31}$$

$$\text{subject to}\begin{cases} W \geq \sum_k \sum_{\mathbf{x}[k]} r_{\mathbf{x}[k],\mathbf{x}_1[k]} B_{\mathbf{x}[k]} + \sum_l \xi \mathbf{x}_1[l] & \forall \mathbf{x}_1 \in Dom(\mathbf{X}_1) \\ C_1 \text{ and } C_2 \end{cases} \tag{32}$$

$$= \min_{\{L_{\mathbf{x}[k]}, X_i, W\}} W$$

$$\text{subject to}\begin{cases} W \geq \sum_k \sum_{\mathbf{x}[k]} R_{\mathbf{x}_1[k]} + \sum_l \xi \mathbf{x}_1[l] & \forall \mathbf{x}_1 \in Dom(\mathbf{X}_1) \\ R_{\mathbf{x}_1[k]} = \sum_{\mathbf{x}[k]} r_{\mathbf{x}[k],\mathbf{x}_1[k]} B_{\mathbf{x}[k]} & \forall k, \mathbf{x}_1 \in Dom(\mathbf{X}_1[k]) \\ C_1 \text{ and } C_2 \end{cases} \tag{33}$$

Equation (31) contains one constraint per state $\mathbf{x}_1$. The addition of the penalty terms $\xi\mathbf{x}_1$ $[l]$ ensures maintenance of the feasibility constraints on $\mathbf{x}_1$; it makes a constraint on W needless when its equivalent state $\mathbf{x}_1$ is infeasible.

4. Discussions

The effectuation of a laudable healthcare delivery depends on the willingness of the DM to make adroit decisions. Their disposition is such that invites diligence and ingenuity in the art of decision-making. Since, as earlier said, they are most often on the horn of a dilemma; they either make the right decision or live in *regret*. While this regret may abide by the maximum, the question of minimizing the regret comes to mind. Does a street-to-street marketer decide to go *al fresco* during the rainy season when rain is anticipated? They must have umbrellas on them. Therefore, since the regret is rather anticipated, it is incumbent upon the DM to make adequate cushioning provisions.

Regret is an inescapable abstract component of healthcare delivery. It is the feeling induced by choosing a preferred decision (state) against any/next better alternative. Regret may be maxi- mum in amplitude; the higher the regret the worse the scenario. It is against this background that the mini-max regret criterion is employed to assuage the deleterious effect of the worst-case scenario in the healthcare delivery system.

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Consent for publication

We declare that we consented for the publication of this research work.

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Asian Journal of Basic Science & Research
Volume 5, Issue 1, Pages 109-118, January-March 2023

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